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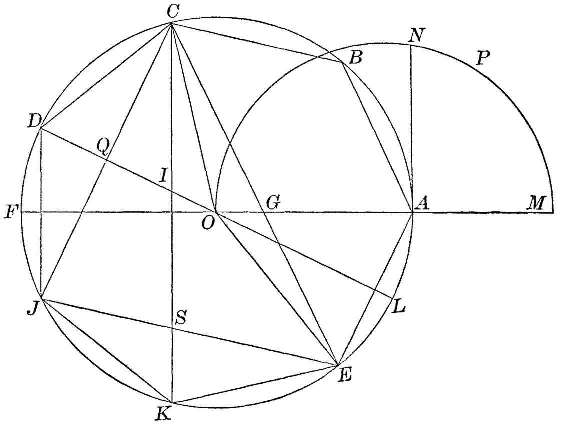
A GEOMETRICAL DISCUSSION OF THE REGULAR INSCRIBED HEPTAGON.

By J. Q. McNatt, Florence, Colo.

To calculate the length of the side of a regular inscribed heptagon when the radius is unity, suppose, in the accompanying figure, that the circumference is divided into seven equal parts at the points A, B, C, D, J, K, and E.



Draw the diameter DL and the chords JC, JE, CK, and CE and let CK cut JE at S.

Let h be the length of the side of the regular inscribed heptagon. It is proposed to compute h in terms of the radius as a unit. We have $DL : DC :: DC : DQ$, or in terms of the radius and h, $2 : h :: h : DQ$. That is, $DQ = h^2/2$. Moreover, $QL \times DQ = QC \times JQ$, and $QC = JQ$, or in terms of h we get

$$JQ = \sqrt{\left(2 - \frac{h^2}{2}\right)\left(\frac{h^2}{2}\right)} = \tfrac{1}{2}h\sqrt{4 - h^2}.$$

(1) Now $JE = JC = h\sqrt{4 - h^2}$, and $KE = SE$ since the two triangles KCE and KES are equiangular and hence similar, and since KCE is isosceles. Then in terms of h we find (2) $JS = JE - SE$, or $JS = h\sqrt{4 - h^2} - h$.

Now let DL cut KC at I and draw the radius CO. Then the two triangles OCD and CDI are equiangular and similar, and OCD is isosceles.

Then $CO : DC :: DC : DI$ or in terms of h, $1 : h :: h : DI$, from which $DI = h^2$. Also $DL - DI = IL = 2 - h^2$. Again, triangle DCI being isosceles, $DC = CI$, or $CI = h$. By geometry $IL \times DI = CI \times IK$, or in terms of h we have,

$$(2 - h^2)(h^2) = (h)(IK), \quad \text{or} \quad IK = 2h - h^3,$$

but $CI = h$, so that $KC = 3h - h^3$.

As the triangles KES and KCE are similar isosceles triangles, we have

$$CK : KE :: KE : SK,$$

or in terms of h,

(3) $$3h - h^3 : h :: h : SK,$$

from which

$$SK = \frac{h}{3 - h^2}.$$

In the isosceles triangle JCS, $CS = JC = h\sqrt{4 - h^2}$ as shown above in (1). By geometry $CS \times SK = JS \times SE$, or in terms of h, using (1), (2) and (3) above,

(4) $$(h\sqrt{4 - h^2})\left(\frac{h}{3 - h^2}\right) = (h\sqrt{4 - h^2} - h)(h).$$

Simplifying equation (4) we find

$$8 - 6h^2 + h^4 = (3 - h^2)\sqrt{4 - h^2}.$$

Factoring, dividing both members by $\sqrt{4 - h^2}$, squaring, and transposing, we get

(5) $$7 - 14h^2 + 7h^4 - h^6 = 0,$$

which we call *The Heptagon Cubic.* Solving (5) by Horner's method we find,

$$h^2 = .7530203962821 \cdots,$$

or

$$h = .86776748 \cdots,$$

which is the side of the regular heptagon inscribed in a circle of radius unity.

APPROXIMATE CONSTRUCTION OF A HEPTAGON.

From the similar isosceles triangles OEA and EAG, we get

(6) $$OE : EA :: EA : GA,$$

or

$$1 : h :: h : GA,$$

from which

$$GA = h^2,$$

but

$$h^2 = .75 +, \text{ or } \tfrac{3}{4}OA.$$

Produce the line FA indefinitely and lay off on it the point G, so that

$$GA = \tfrac{3}{4}OA.$$

Then mark the point M, so that $AM = GA$.

With OM as a diameter construct the semicircle OPM and at the point A erect a perpendicular AN, which will be a mean proportional between OA and AM, that is, between OE and GA, so that by (6) $AN = EA$.

Then with a chord equal to the line AN lay off the points of the seven sided polygon, A, B, C, D, etc., which will be a regular inscribed heptagon, to a very close approximation.